General instructions for Students: Whatever be the notes provided, everything must be copied in the Maths copy and then do the HOMEWORK in the same copy.

$CLASS - IX$

$MATHEMATICS$

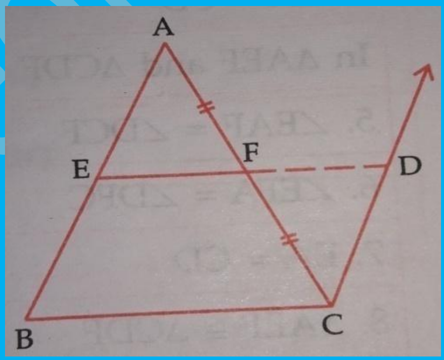
$11.\ Mid-Point\ Theorem\ \ (Part-I)$

$Mid-Point\ Theorem$

$Statement$: $The\ line\ segment\ joining\ the\ mid-points\ of\ any\ two\ sides\ of\ a\ triangle$

$is\ parallel\ to\ the\ third\ side\ and\ is\ equal\ to\ half\ of\ it.$

$Given:\ \ In\ \triangle ABC,\ E\ and\ F\ are\ the\ mid-points\ of\ sides\ AB\ and\ AC\ respectively.$



$To\ prove:\ \ EF\ \parallel BC\ \ and\ \ EF = \frac{1}{2}BC.$

$Construction: Draw\ CD\ \parallel BA, meeting\ EF\ produced\ at\ D.$

$Proof:\ \ In\ \triangle AEF\ and\ \triangle CDF$

$$AF = CF \qquad (Given)$$

$$\angle AFE = \angle CFD \qquad (Vert. opp. \angle s)$$

$$\angle EAF = \angle DCF \qquad (Alt. int. \angle s)$$

$$\triangle AEF \cong \triangle CDF \qquad (ASA\ congruence\ rule)$$

$$AE = CD\ \ and\ \ EF = FD \qquad (C.P.C.T.)$$

$But,\qquad AE = EB$

$\therefore\qquad CD = EB\ and\ CD \parallel BA \qquad (const.)$

$\therefore\qquad BCDE\ is\ a\ parallelogram.$

$\therefore\qquad EF \parallel BC\ \ and\ \ ED = BC$

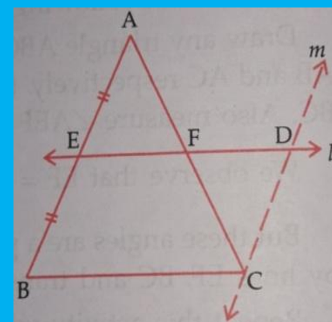
$$EF = \frac{1}{2}ED$$

$$EF = \frac{1}{2}BC$$

$Hence,\ \ EF \parallel BC\ \ and\ \ EF = \frac{1}{2}BC.\qquad Proved.$

Converse of Mid – Point Theorem

Statement : *The line drawn through the mid – point of one side of a triangle parallel to another side bisects the third side.*



In $\triangle ABC$, E is the mid – point of side AB.

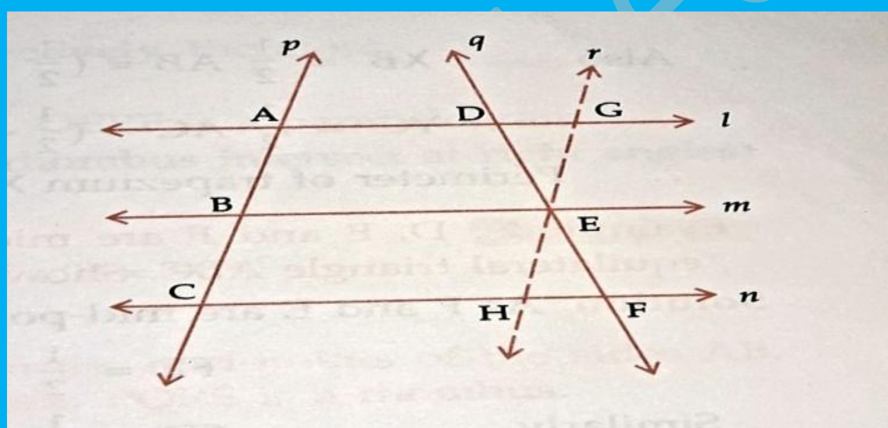
Line l drawn through E and parallel to BC meeting AC at F.

$\therefore$ EF bisects AC $\Rightarrow$ F is the mid – point of AC

i.e. $AF = FC$.

Theorem on intercepts

Statement : *If a transversal makes equal intercepts on three (or more) parallel lines, then any other line cutting them also makes equal intercepts.*



Three parallel lines l, m and n are cut by a transversal p at A, B and C respectively such that $AB = BC$.

Also q is another transversal cutting l, m and n at D, E and F respectively.

Through E, draw a line r parallel to p, meeting l, m and n at G, E and H respectively.

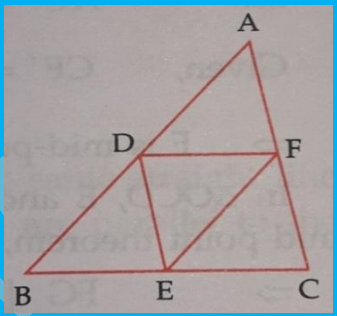
$\therefore$ E is the mid – point of DF

i.e. $DE = EF$

Q.No. 2 *Prove that the four triangles formed by joining in pairs of mid – points of the sides of a triangle are congruent to each other.*

Solution : Let D, E and F be the mid – points of sides AB, BC and AC respectively.

$DF \parallel BC$ and $DF = \frac{1}{2} BC$ (Mid – point Theorem) ……… (i)



$\Longrightarrow DF \parallel BE$ and $DF = BE$

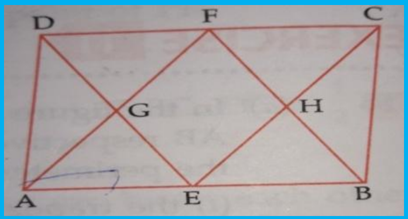
$\therefore$ $BEFD$ is a parallelogram

$\triangle BED \cong \triangle DEF$

[DE is a diagonal of a $\parallel gm$ $BEFD$.
The diagonal of a parallelogram divides into two congruent triangles.]

Similarly, $\triangle ADF \cong \triangle DEF$ and $\triangle CEF \cong \triangle DEF$ Proved.

Q.No. 8 *In the adjoining figure, ABCD is a parallelogram. E and F are mid – points of the sides AB and CD respectively. The straight lines AF and BF meet the straight lines ED and EC in points G and H respectively. Prove that (i) $\triangle HEB \cong \triangle HCF$ (ii) GEHF is a parallelogram*



Solution : In $\triangle HEB$ and $\triangle HCF$, $\angle BHE = \angle CHF$ (Vert. opp. $\angle s$)

$\angle HEB = \angle HCF$ (Alt. int. $\angle s$)

$EB = FC$ ($\because AB = DC$ and E and F are mid – points of the sides AB and CD)

$\triangle HEB \cong \triangle HCF$ (AAS congrucnce rule) Proved.

$HB = HF$ (C.P.C.T.)

Similarly, $GA = GF$

In $\triangle ABF$, $GE \parallel FB$ and $GE = \frac{1}{2} FB$ [Mid – point theorem]

$\Longrightarrow$ $GE \parallel FH$ and $GE = FH$ [$FH = \frac{1}{2} FB$]

$\therefore$ $GEHF$ is a parallelogram Proved.

HOMEWORK

EXERCISE – 11
QUESTION NUMBERS : 1 (a), (c); 3, 4, 5 and 7